Parity violation in $n + ^3He \rightarrow ^3H + p$ reaction: resonance approach.

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Abstract

The method based on microscopic theory of nuclear reactions has been applied for the analysis of parity violating effects in a few-body systems. Different parity violating and parity conserving asymmetries and their dependence on neutron energy have been estimated for $n + ^3He \rightarrow ^3H + p$ reaction. The estimated effects are in a good agreement with available exact calculations.

INTRODUCTION

The study of parity violating (PV) effects in low energy physics is important for the understanding of main features of the Standard model and for the possible search for manifestation of new physics. During the last decades, many calculations of different experimentally observed PV effects in nuclear physics have been done. However, in the last years, it became clear (see, for example [1–4] and references therein) that the traditional DDH [5] method for the calculation of PV effects cannot reliably describe the available experimental data. This could be blamed on the "wrong" experimental data, however it might be that the DDH approach is not adequate for the description of the set of precise experimental data because it is based on a number of models and assumptions. To resolve this discrepancy, it is desirable to increase the number of experimental data for different PV parameters in a few-body systems. The calculations of nuclear related effects for these systems can be done with high precision, eliminating as many as possible nuclear model dependent factors involved in PV effects. Unfortunately, currently available data of experimentally measured PV effects in these systems are not enough to constrain all parameters required for calculations, therefore, any new potentially possible measurement is very important. Since PV effects in a few-body systems are usually very small and precise calculations of them are rather difficult, it is desirable to have a method for a reliable estimate of possible observable parameters using available experimental data. This will give the opportunity to choose the right system and right PV observables for new experiments.

Recently, it has been proposed to measure PV asymmetry of protons in $n+^3He \to^3 H+p$ reaction with polarized neutrons at the Spallation Neutron Source at the Oak Ridge National Laboratory. The ^3He and ^4He systems were subjects of intensive investigation for a long time, and as a result, many parameters related to reactions with neutrons and protons, as well as to excitation energy levels of these nuclei, have been measured and evaluated by a number of different groups. This rather comprehensive data provides the opportunity to estimate values of possible PV effects and their dependence on neutron energy in $n+^3He \to^3 H+p$ reaction using microscopic nuclear reaction theory approach.

DESCRIPTION OF PARITY VIOLATING EFFECTS

Let us consider the $n +^3 He \to^3 H + p$ reaction with low energy neutrons. For neutron energy $E_n \sim 0.01\, eV$, which corresponds to a wave vector $k_n \sim 2.19 \cdot 10^{-5}\, fm^{-1}$, the energy of outgoing protons and proton wave vector are $E_p = 0.764\, MeV$ and $k_p = 0.19\, fm^{-1}$, correspondingly. Taking a characteristic ^3He radius as $R = 1.97\, fm$, one obtains $(k_n R) \sim 4 \cdot 10^{-4}$ and $(k_p R) \sim 0.4$. Therefore, for the initial channel, contributions from p-wave neutrons to a reaction matrix (amplitude) are highly suppressed, whereas for the final channel, the amplitude with orbital momenta of protons $l = 0$ and $l = 1$ have the same order of magnitude. The contribution from d-wave protons is suppressed by a factor ~ 0.025, therefore, one can ignore d-waves within the accuracy of our estimates. Assuming that neutrons, as well as ^3He nuclei, may have a polarization, one shall consider four parity violating (PV) and four parity conserving (PC) angular correlations shown in Table I, where $\vec{\sigma}$ and $\vec{I}$ are neutron and nuclear spins. (It is important to know the values of PC correlations because they usually are one of the main sources of experimental errors for PV effects, and also because they can be rather easily measured and, as a consequence, can serve as an indirect proof of the correctness of calculations of PV effects.) We will focus on PV correlation $(\vec{\sigma} \cdot \vec{k}_p)$ which leads to PV asymmetry α_{PV} of outgoing protons in a direction along to neutron polarization and opposite to it. It can be seen that the asymmetry related to the $(\vec{I} \cdot \vec{k}_p)$ correlation has exactly the same value for this reaction. For the completeness of the consideration, we will also calculate the PV effect related to differences of total cross sections $\sigma^{tot}_{\pm}$ for neutrons with opposite helicities when they propagate through ^3He target, $P = (\sigma^{tot}_{+} - \sigma^{tot}_{-})/(\sigma^{tot}_{+} + \sigma^{tot}_{-})$, which is related to the $(\vec{\sigma} \cdot \vec{k}_n)$ correlation. The corresponding difference of total cross sections for a propagation of unpolarized neutrons through polarized target, the $(\vec{I} \cdot \vec{k}_n)$ correlation, has the same value P in our case. It should be noted that the $(\vec{\sigma} \cdot \vec{k}_n)$ correlation leads also

TABLE I. Possible parity violating and four parity conserving angular correlations.

PV	PC
$(\vec{\sigma} \cdot \vec{k}_p)$	$(\vec{k}_n \cdot \vec{k}_p)$
$(\vec{\sigma} \cdot \vec{k}_n)$	$(\vec{\sigma} \cdot [\vec{k}_n \times \vec{k}_p])$
$(\vec{I} \cdot \vec{k}_p)$	$(\vec{\sigma} \cdot \vec{I})$
$(\vec{I} \cdot \vec{k}_n)$	$(\vec{I} \cdot [\vec{k}_n \times \vec{k}_p])$

to PV neutron spin precession around the direction of neutron momentum. However, the angle of precession is very small (about $10^{-9} - 10^{-10}$ rad for thermal neutrons, and it can reach the value up to about 10^{-5} rad for $E_n \sim 0.5$ MeV for the target of the size of neutron mean free path), therefore, we will not consider it here. In addition, we will calculate the left-right asymmetry α_{LR} which corresponds to the PC correlation $(\vec{\sigma} \cdot [\vec{k}_p \times \vec{k}_p])$, because it could be a source of systematic effects in the measurement of α_{PV}.

Using standard techniques (see, for example [6]), one can represent these asymmetries in terms of matrix $\hat{R}$ which are related to reaction matrix $\hat{T}$ and to S-matrix as:

$$\hat{R} = 2\pi i \hat{T} = \hat{1} - \hat{S} \tag{1}$$

Then, for our case

$$\begin{aligned} \alpha_{PV} = \frac{2}{r} \mathrm{Re}[& -3\sqrt{2} < 01|R^1|10 > \cdot < 00|R^0|00>^* \\ & + (\sqrt{6} < 11|R^0|00 > +6 < 11|R^1|10 >) < 10|R^1|10>^*] \end{aligned} \tag{2}$$

and

$$\begin{aligned} \alpha_{LR} = \frac{1}{r} Im[& 6\sqrt{3} < 10|R^1|11 > \cdot < 00|R^0|00>^* + 6\sqrt{3} < 11|R^1|01 > \cdot < 10|R^1|10>^* \\ & + 3(2 < 11|R^0|00 > +\sqrt{6} < 11|R^1|10 >) < 10|R^1|11>^* \\ & + 6\sqrt{2} < 00|R^0|11 > \cdot < 01|R^1|10>^* + 6\sqrt{3} < 10|R^1|01 > \cdot < 11|R^1|10>^* \\ & + \sqrt{6} < 10|R^1|10 > (2 < 11|R^0|11>^* + 3 < 11|R^1|11>^*) \\ & + 5\sqrt{6} < 11|R^2|11 > \cdot < 10|R^1|10>^*], \end{aligned} \tag{3}$$

where

$$r = (| < 00|R^0|00 > |^2 + 3| < 10|R^1|10 > |^2). \tag{4}$$

We use spin-channel representation, where for the matrix element $< s'l'|R^J|sl >$, l and l' are orbital momenta of initial and final channels with corresponding spin-channels s and s', and J is the total spin of the system. For a transmission type of observable P, one obtains:

$$\begin{aligned} P = & -\frac{\mathrm{Re}[< 00|R^0|11 > + < 11|R^0|00 >]}{\mathrm{Re}[< 00|R^0|00 > +3 < 10|R^1|10 >]} \\ & + \sqrt{3}\frac{\mathrm{Re}[< 10|R^1|01 > + < 01|R^1|10 >]}{\mathrm{Re}[< 00|R^0|00 > +3 < 10|R^1|10 >]} \\ & + \sqrt{6}\frac{\mathrm{Re}[< 10|R^1|11 > + < 11|R^1|10 >]}{\mathrm{Re}[< 00|R^0|00 > +3 < 10|R^1|10 >]}. \end{aligned}$$

Calculations of matrix elements $< s'l'|R^J|sl >$ for parity violating effects in nuclear reactions have been done [6] using distorted wave Born approximation in microscopic theory of nuclear reactions [7]. They lead to the symmetry violating amplitudes induced by parity violating potential W

$$R_{PV}^{fi} = 2\pi i < \Psi_f^- |W| \Psi_i^+ >, \tag{5}$$

where $\Psi_{i,f}^{\pm}$ are the eigenfunctions of the nuclear P-invariant Hamiltonian with the appropriate boundary conditions [7]:

$$\Psi_{i,f}^{\pm} = \sum_k a_{k(i,f)}^{\pm}(E)\ \phi_k + \sum_m \int b_{m(i,f)}^{\pm}(E,E')\ \chi_m^{\pm}(E')\ dE'. \tag{6}$$

Here, ϕ_k is the wave function of the k^{th} resonance and $\chi_m^{\pm}(E)$ is the potential scattering wave function in the channel m. The coefficient

$$a_{k(i,f)}^{\pm}(E) = \frac{\exp\left(\pm i\delta_{i,f}\right)}{(2\pi)^{\frac{1}{2}}} \frac{(\Gamma_k^{i,f})^{\frac{1}{2}}}{E - E_k \pm \frac{i}{2}\Gamma_k} \tag{7}$$

describes nuclear resonances contributions, and the coefficient $b_{m(i,f)}^{\pm}(E,E')$ describes potential scattering and interactions between the continuous spectrum and resonances. (Here, E_k, Γ_k, and Γ_k^i are the energy, the total width, and the partial width in the channel i of the k-th resonance, E is the neutron energy, and δ_i is the potential scattering phase in the channel i; $(\Gamma_k^i)^{\frac{1}{2}} = (2\pi)^{\frac{1}{2}} < \chi_i(E)|V|\phi_k >$, where V is a residual interaction operator.) As it was shown in [6] for nuclei with rather large atomic numbers, the resonance contribution is dominant. Then, for the simplest case with only two resonances with opposite parities, the expressions for matrix element $\hat{R}$ for neutron-proton reaction with parity violation is:

$$< s'l'|R^J|sl > = -\frac{iw(\Gamma_l^n(s)\Gamma_{l'}^p(s'))^{\frac{1}{2}}}{(E - E_l + i\Gamma_l/2)(E - E_{l'} + i\Gamma_{l'}/2)} e^{i(\delta_l^n + \delta_{l'}^p)}, \tag{8}$$

and with conservation of parity (one resonance contribution) is:

$$< s'l'|R^J|sl > = \frac{i(\Gamma_l^n(s)\Gamma_{l'}^p(s'))^{\frac{1}{2}}}{(E - E_l + i\Gamma_l/2)} e^{i(\delta_l^n + \delta_{l'}^p)}, \tag{9}$$

where $w = -\int \phi_l W \phi_{l'} d\tau$ is parity violating nuclear matrix element mixing parities of two resonances. The above $\hat{R}$ matrix elements could be represented by diagrams shown on Fig.(1). Thus, PV asymmetry α_{PV} is proportional to the real part the product of $\hat{R}$ matrices presented by diagrams a and c, and PC asymmetry α_{LR} is proportional to the imaginary part of the product of $\hat{R}$ matrices presented by diagrams a and b (see for details [6]).

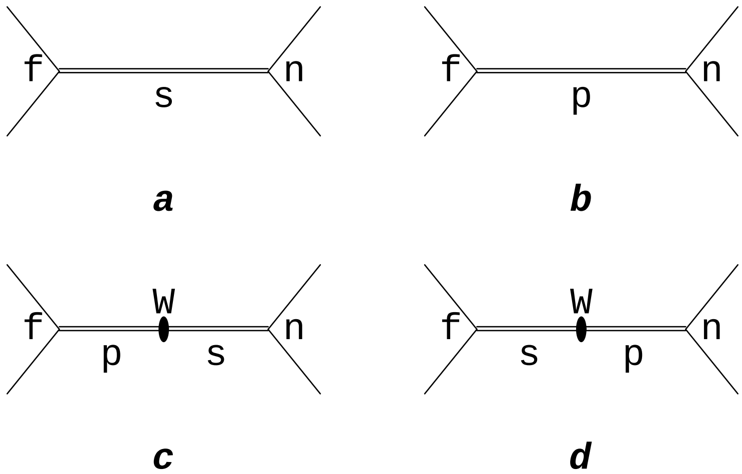

FIG. 1. Parity conserving (diagrams a and b) and parity violating (diagrams c and d) matrix elements of $\hat{R}$. Symbols s and p corresponds to s- and p-wave neutrons for the initial (neutron) channel, and indicate parity of the final channel.

This technique has been proven to work very well for calculation of nuclear PV effects for intermediate and heavy nuclei. Assuming the dominant resonance contribution to PV effects for $n+^3He \rightarrow ^3H+p$ reaction, we will apply this approach to estimate characteristic values of PV effects using parametrization of PV effects in terms of known resonance structure of the system. Fortunately, the detailed structure of resonances (^4He levels) [8] and low energy neutron scattering parameters [9] are well known for this reaction from numerous experiments.

To estimate PV and PC asymmetries in $n+^3He \rightarrow ^3H+p$ reactions using the described above formalism, we will take into account all known resonances [8, 9] which result in multi-resonance representation for $\hat{R}$ matrix elements. From the selection rules for angular momenta (see Eqs. (2), (3), (5) and general expressions in [6]), one can see that for low energy neutrons only resonances with the total spin of $J = 0, 1$ contribute to PV asymmetries of the interest. However, for the left-right PC asymmetry, we have to consider $J = 0, 1, 2$. Thus, for PV effects we consider contributions from nine low energy resonances [8, 9] (see Table (II)): one resonance with total angular momentum and parity $J^\pi = 0^+$, three with $J^\pi = 0^-$, four with $J^\pi = 1^-$, and one with $J^\pi = 1^+$. For further calculations, we assume that all weak matrix elements, which mix resonances with opposite parities, have the same values and are described by a phenomenological formula [6] $w = 2 \cdot 10^{-4} eV \sqrt{\bar{D}(eV)}$ (where $\bar{D}$ is

an average energy level spacing). This formula is in good agreement with other statistical nuclear model estimates [10–12] of nuclear weak matrix elements for medium and heavy nuclei. The extrapolation of this formula to the region of one-particle nuclear excitation leads to correct value for weak nucleon-nucleon interaction. Therefore, one can use this approximation for rough estimates of average values of weak matrix elements in few-body systems. This leads to the value of weak matrix element $w = 0.5\ eV$ (with $\bar{D} \simeq 6\ MeV$), which is rather close to the typical value of one particle weak matrix element. One can see from Eqs.(8) and (9) that the expressions for PV and PC $\hat{R}$ matrices depend not on neutron and proton partial widths but on their amplitudes, the values of which depend on particular spin-channels. Since we know only partial widths, we have to make assumptions about values of amplitudes of partial widths for a specific spin-channel and about their signs (phases). This leads to another uncertainty in our estimation in addition to the given above assumption about weak matrix elements. To treat the spin-channel dependence of partial width amplitudes, we assume that partial widths for each spin-channel are equal to each other. This gives us an average factor of uncertainly of about 2. The signs of width amplitudes, as well as the signs of weak matrix elements w, are left undetermined (random). This also can lead to a factor of uncertainly of 2 or 3. Therefore, one can see that the uncertainly of our multi-resonance calculations is about of one order of magnitude.

DISCUSSION OF RESULT OF CALCULATIONS

Taking into account the considerations given above and using resonance parameters [8, 9] of the Table (II), one can estimate the PV asymmetry for thermal neutrons as

$$\alpha_{PV} = -(1-4) \cdot 10^{-7}. \tag{10}$$

The set of resonance parameters of the Table (III) results in a slightly lager PV asymmetry

$$\alpha_{PV} = -(4-8) \cdot 10^{-7}. \tag{11}$$

The difference of these two sets is related to the discrepancy between [9] and [8] for resonance parameters for the first positive resonance ($E_n = 0.430\ MeV$).

The left-right asymmetry at thermal energy is less sensitive to the parameters of this first resonance and has about the same value for these two sets of resonance parameters

$$\alpha_{LR} = -(2-8) \cdot 10^{-4}. \tag{12}$$

It should be noted that for the calculation of the left-right asymmetry, we add first three 2^- resonances [8] which do not contribute to PV effects for low energy neutrons.

The value of PV in neutron transmission for the first choice of parameters is

$$P = -(2-4) \cdot 10^{-10}, \tag{13}$$

and for the second choice is

$$P = -(0.8-1.6) \cdot 10^{-10}. \tag{14}$$

One can see that the parameter P is very small for neutrons with thermal energy, but it is essentially enhanced in a few-MeV region (see Fig.(2). The asymmetry α_{PV} also shows a resonance behavior but its enhancement is not very large (see Fig.(3)).

To show contributions of each resonance to PV asymmetry α_{PV} and to transmission parameter P, we normalized contributions from each resonance in terms of relative intensity to the most strong one, which is taken as 100% (see last two columns in Tables (II) and (III)). Some resonances contribute through two different spin-channels $s = 0$ and $s = 1$. In those cases, the contributions from two spin-channels can be either with the same sign or with the opposite sign, depending on unknown phases of amplitudes of partial widths and weak matrix elements (see, for example, resonance at 3.062 MeV in Table (II)). As can be seen from these tables, different resonances contribute essentially differently to the value of PV violating effects (this is also correct for parity conserving asymmetries). Moreover, different sets of resonance parameters can change weights of the resonances for a particular asymmetry. For example, the lowest 0^--resonance contribution to the asymmetry α_{PV} appears to be 3% using the parameters of Table (II), while it would be dominant one using the parameters of Table (III). This is related to the fact that for the set of Table (II) the contribution of the 0^--resonance to the α_{PV} is suppressed by a factor of about 40 due to destructive interference between parity conserving and parity violating amplitudes. Therefore, the readability of this method can be essentially improved by increasing the accuracy in measurements of parameters of the most "important" resonances.

It should be noted that the estimated value of the PV asymmetry α_{PV} at thermal energy (see Eqs. (10) and (11)) is surprisingly in very good agreement with exact calculations for zero energy neutrons [13]. This could be considered as an additional argument for reliability of the suggested resonance approach. Also, matching the estimated value of the observable

TABLE II. Resonance parameters (Set 1). Here E_r is a resonance energy; T and J^π are resonance isospin and the total resonance spin with parity; Γ and Γ_p are total and proton widths; Γ_n, Γ_n^0 and l are neutron width, reduced width, and angular momentum, correspondingly; α_{PV} (%) and P (%) are normalized contribution of the resonance to α_{PV} and P, correspondingly.

$E_r(MeV)$	J^π	l	T	$\Gamma_n(MeV)$	$\Gamma_n^0(eV)$	$\Gamma_p(MeV)$	$\Gamma(MeV)$	α_{PV} (%)	P (%)
-0.211	0+	0	0		954.4	1.153	1.153		
0.430	0-	1	0	0.48		0.05	0.53	3.1	100
3.062	1-	1	1	2.76		3.44	6.20	100±26	2±1
3.672	1-	1	0	2.87		3.08	6.10	75±24	1±1
4.702	0-	1	1	3.85		4.12	7.97	20	3
5.372	1-	1	1	6.14		6.52	12.66	79±18	1
7.732	1+	0	0	4.66		4.725	9.89		
7.792	1-	1	0	0.08		0.07	3.92	2±1	0
8.062	0-	1	0	0.01		0.01	4.89	14	0

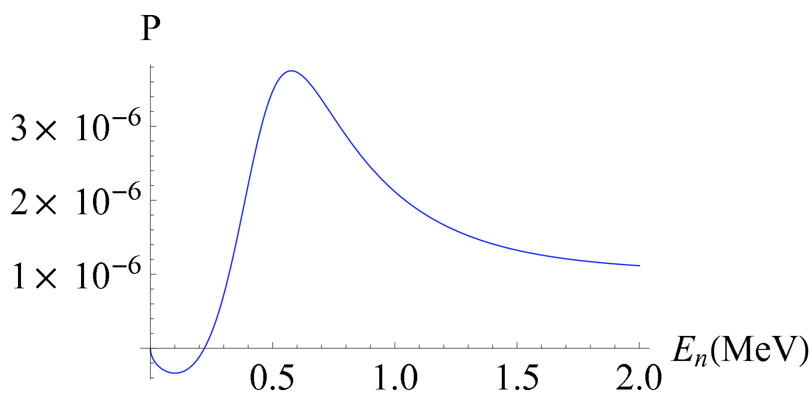

FIG. 2. (Color online) Resonance enhancement of of the P parameter.

parameter with exact calculations at low energy gives us the opportunity to predict PV effects in a wide range of neutron energies.

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TABLE III. Resonance parameters (Set 2). Here E_r is a resonance energy; T and J^π are resonance isospin and the total resonance spin with parity; Γ and Γ_p are total and proton widths; Γ_n, Γ_n^0 and l are neutron width, reduced width, and angular momentum, correspondingly; α_{PV} (%) and P (%) are normalized contribution of the resonance to α_{PV} and P, correspondingly.

$E_r(MeV)$	J^π	l	T	$\Gamma_n(MeV)$	$\Gamma_n^0(eV)$	$\Gamma_p(MeV)$	$\Gamma(MeV)$	α_{PV} (%)	P (%)
-0.211	0+	0	0		954.4	1.153	1.153		
0.430	0-	1	0	0.20		0.640	0.84	100	100
3.062	1-	1	1	2.76		3.44	6.20	82±27	4±3
3.672	1-	1	0	2.87		3.08	6.10	62±20	3±2
4.702	0-	1	1	3.85		4.12	7.97	16	8
5.372	1-	1	1	6.14		6.52	12.66	65±15	2±1.5
7.732	1+	0	0	4.66		4.725	9.89		
7.792	1-	1	0	0.08		0.07	3.92	2±1	0
8.062	0-	1	0	0.01		0.01	4.89	1	0.5

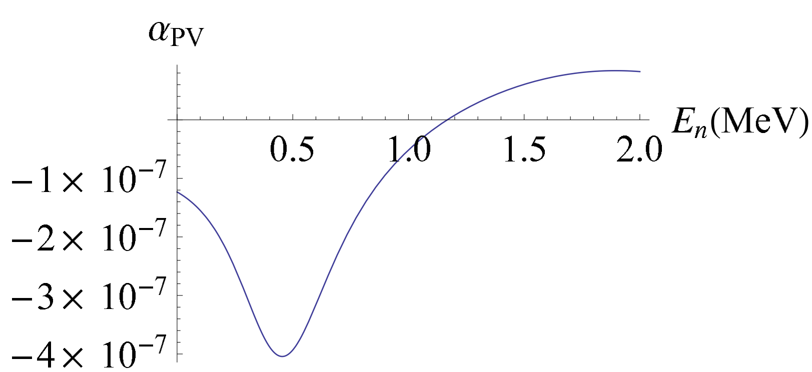

FIG. 3. (Color online) Resonance enhancement of of the α_{PV} asymmetry (for the first set of parameters).

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